## Unit 1, Lesson 16

## Parallel Lines and the Angles in a Triangle

Let's see why the angles in a triangle add to 180 degrees.

### 16.1 True or False: Computational Relationships

Is each equation true or false?
$62-28=60-30$
$3 \cdot-8=(2 \cdot-8)-8$
$\frac{16}{-2}+\frac{24}{-2}=\frac{40}{-2}$

### 16.2 Angle Plus Two

Interactive digital version available

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a.openup.org/ms-math/en/s/ccss-8-1-16-2
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Here is triangle $A B C$.


1. Rotate triangle $A B C 180^{\circ}$ around the midpoint of side $A C$. Label the new vertex D.
2. Rotate triangle $A B C 180^{\circ}$ around the midpoint of side $A B$. Label the new vertex E.
3. Look at angles $E A B, B A C$, and $C A D$. Without measuring, write what you think is the sum of the measures of these angles. Explain or show your reasoning.
4. Is the measure of angle $E A B$ equal to the measure of any angle in triangle $A B C$ ? If so, which one? If not, how do you know?
5. Is the measure of angle $C A D$ equal to the measure of any angle in triangle $A B C$ ? If so, which one? If not, how do you know?
6. What is the sum of the measures of angles $A B C, B A C$, and $A C B$ ?

### 16.3 Every Triangle in the World

Here is $\triangle A B C$. Line $D E$ is parallel to line $A C$.


1. What is $m \angle D B A+b+m \angle C B E$ ? Explain how you know.
2. Use your answer to explain why $a+b+c=180$.
3. Explain why your argument will work for any triangle: that is, explain why the sum of the angle measures in any triangle is $180^{\circ}$.

## Are you ready for more?

1. Using a ruler, create a few quadrilaterals. Use a protractor to measure the four angles inside the quadrilateral. What is the sum of these four angle measures?
2. Come up with an explanation for why anything you notice must be true (hint: draw one diagonal in each quadrilateral).

### 16.4 Four Triangles Revisited

This diagram shows a square $B D F H$ that has been made by images of triangle $A B C$ under rigid transformations.


Given that angle $B A C$ measures 53 degrees, find as many other angle measures as you can.

## Lesson 16 Summary

Using parallel lines and rotations, we can understand why the angles in a triangle always add to $180^{\circ}$. Here is triangle $A B C$. Line $D E$ is parallel to $A C$ and contains $B$.


A 180 degree rotation of triangle $A B C$ around the midpoint of $A B$ interchanges angles $A$ and $D B A$ so they have the same measure: in the picture these angles are marked as $x^{\circ}$. A 180 degree rotation of triangle $A B C$ around the midpoint of $B C$ interchanges angles $C$ and $C B E$ so they have the same measure: in the picture, these angles are marked as $z^{\circ}$. Also, $D B E$ is a straight line because 180 degree rotations take lines to parallel lines. So the three angles with vertex $B$ make a line and they add up to $180^{\circ}(x+y+z=180)$. But $x, y, z$ are the measures of the three angles in $\triangle A B C$ so the sum of the angles in a triangle is always $180^{\circ}$ !

